ISASP IOWA STATEWIDE ASSESSMENT of STUDENT PROGRESS

Using Your District ISASP Data

Mathematics – Performance Changes

Prepared by Iowa Testing Programs

Mathematics – 2022 Statewide Performance Changes

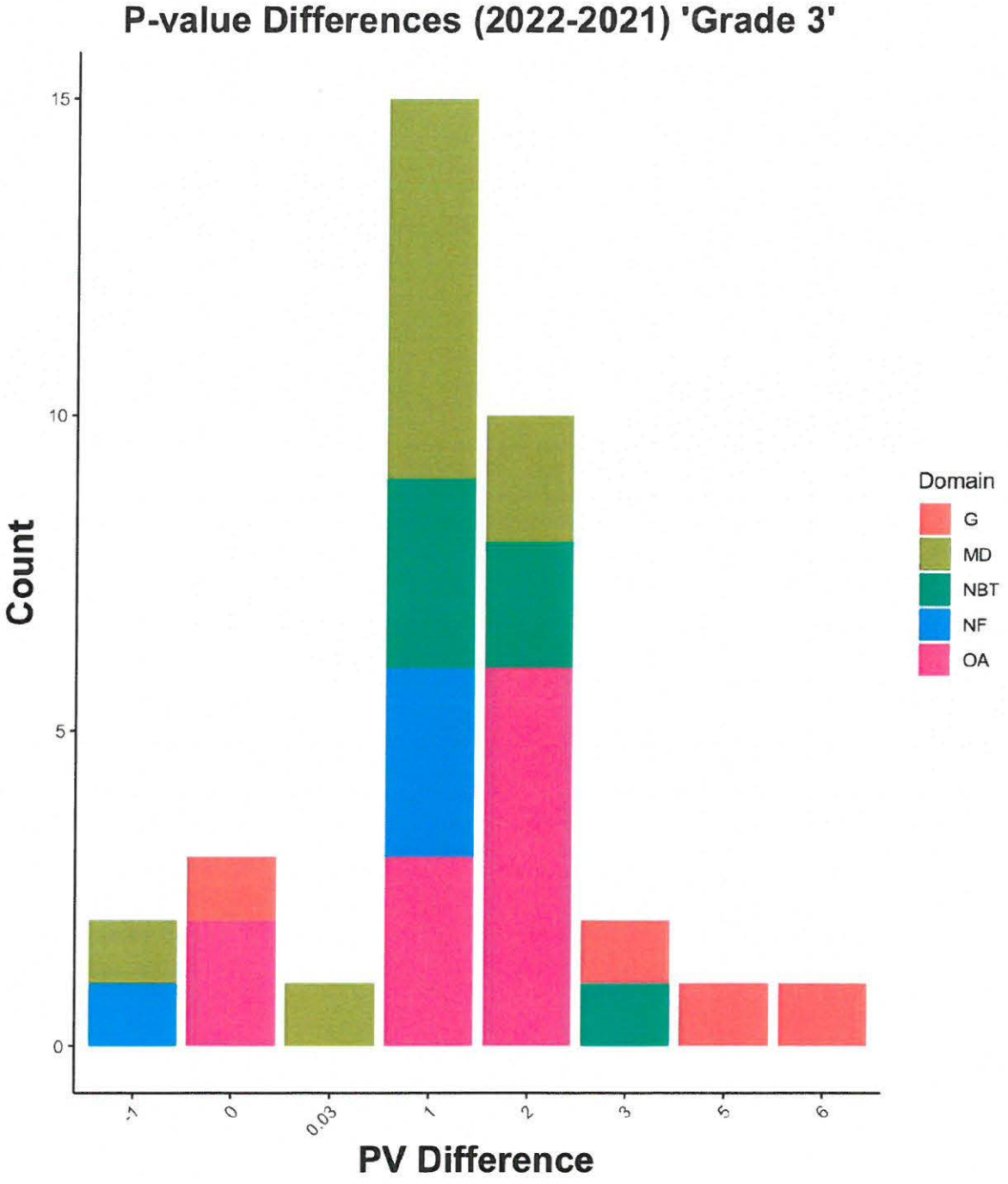
The purpose of this document is to provide descriptive information about performance changes in mathematics between 2021 and 2022 for the Iowa Statewide Assessment of Student Progress (ISASP) in mathematics. The test administrations in 2021 and 2022 were the two occasions for statewide data collections after the COVID-19 pandemic, and they are seen by many as important in understanding specific instances of learning loss or recovery at the state level.

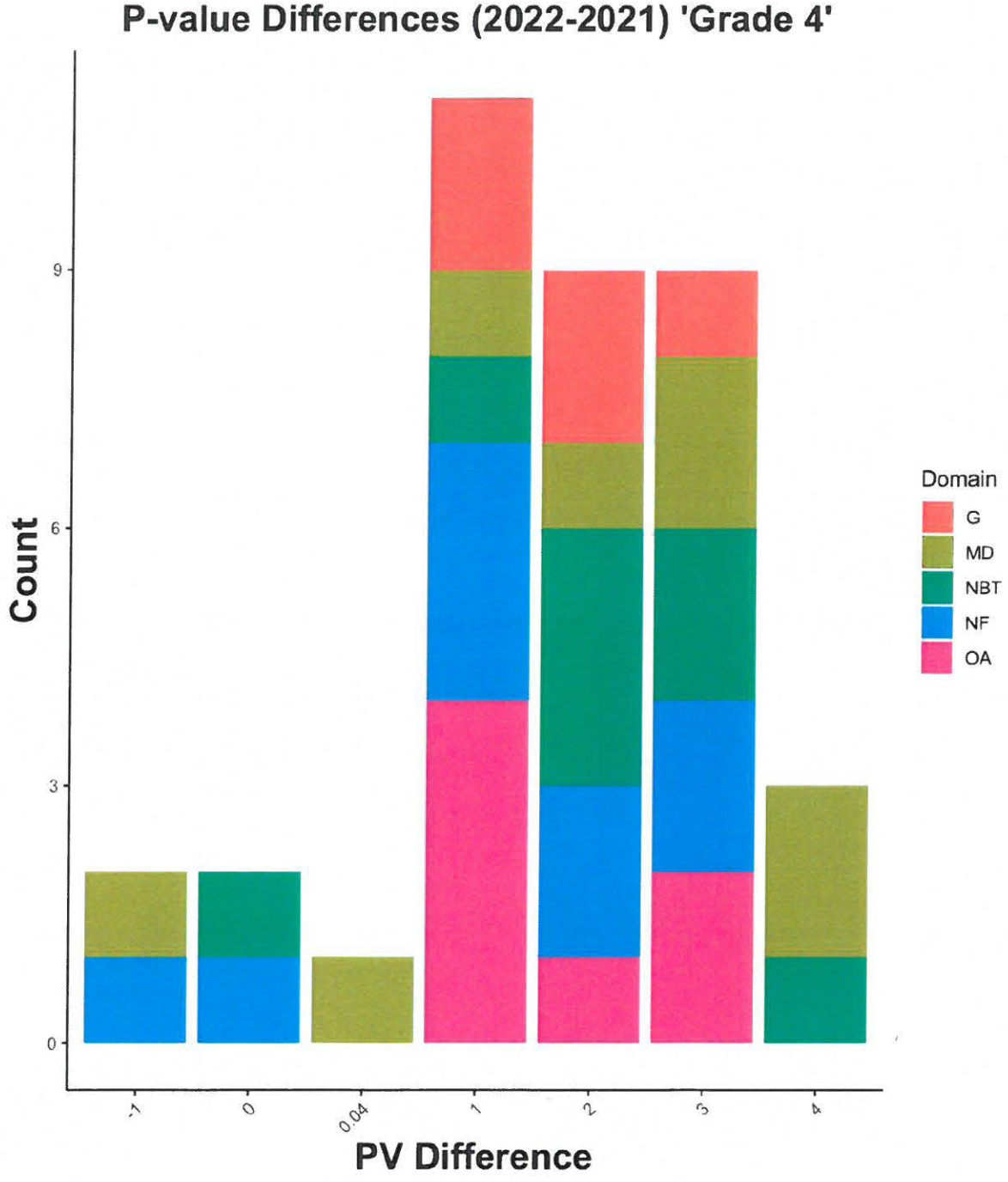
In the spring of 2021, approximately 95 percent of all Iowa students in grades 3 through 11 took proctored exams in the ISASP program. In mathematics, statewide comparisons with previous results indicated a systematic drop in performance at all grade levels (see https://iowa.pearsonaccess.com/resources/research/ISASP_Interpreting%20Mathematics%20Performance%20During%20the%20Pandemic.pdf for a detailed report). As seen in the tables below, mathematics results in 2022 in terms of both average scale score and percent proficient/advanced showed modest signs of learning recovery in grades 3 to 5, mixed results in grades 6 to 8, and continued signs of learning loss in grades 9 to 11.

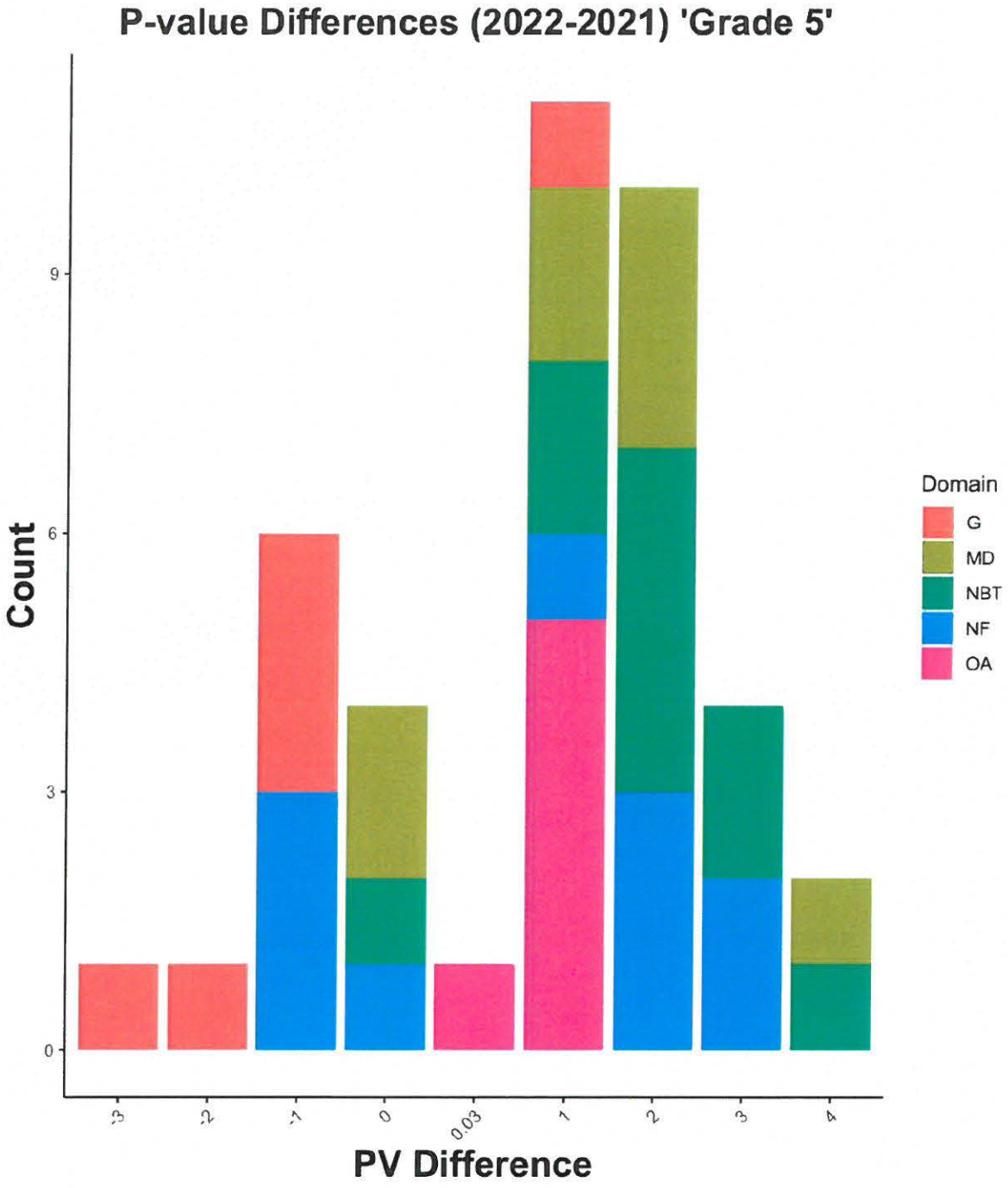
ISASP Average Scale Scores									
Year	Grade								
	3	4	5	6	7	8	9	10	11
2022	411.9	431.0	450.7	474.9	494.4	519.2	533.9	560.4	587.0
2021	410.3	428.1	448.6	473.2	492.6	521.0	536.6	562.9	593.3

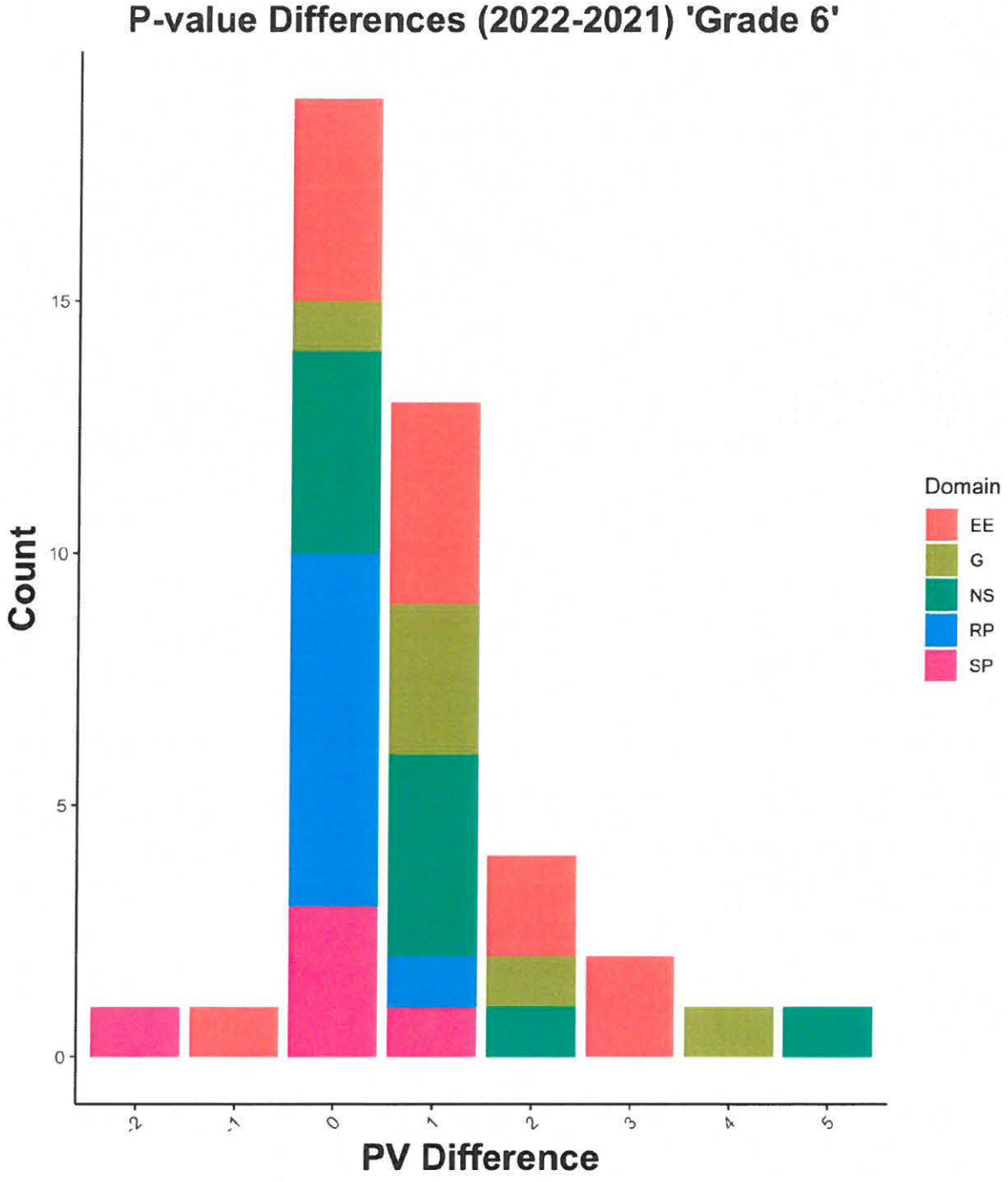
ISASP Percent Proficient and Advanced									
Year	Grade								
	3	4	5	6	7	8	9	10	11
2022	73	69	65	67	65	67	58	61	60
2021	71	67	63	67	64	69	61	64	65

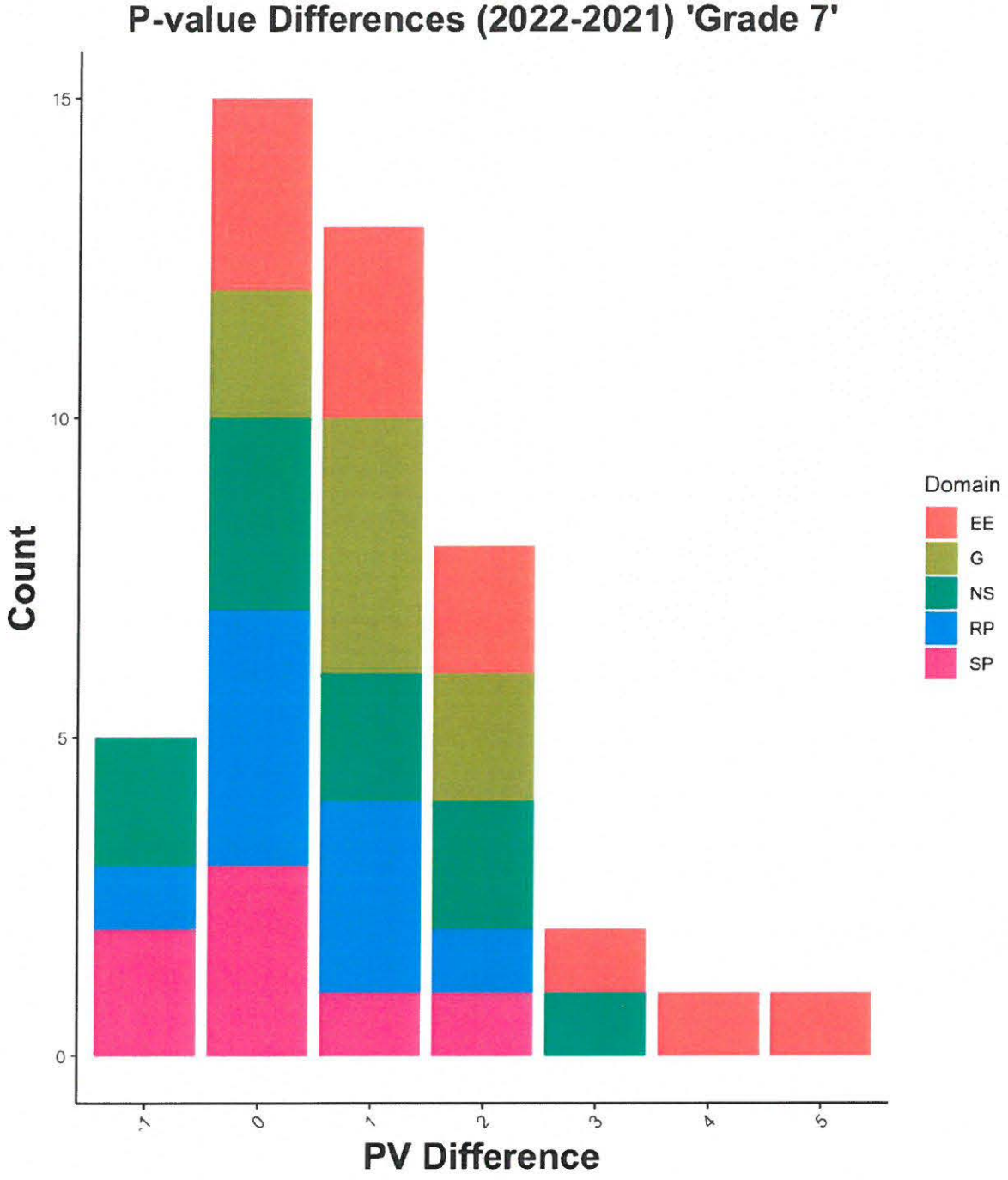
Of further interest with the 2022 results now available are the mathematics domains and Iowa Core Standards that continue to show learning loss, that is, content areas in which 2022 performance was *lower* than 2021 performance, even in grades where the overall results improved. The bar graphs in this document identify, by content domains of the Iowa Core, changes in performance at the item level in each grade in the percent correct metric. The bars are color-coded by Iowa Core domain. Negative differences are signs of learning loss, and positive differences learning gain. Also included is a table of grade-by-grade Iowa Core Standards in mathematics in which there were continued signs of learning loss in 2022.

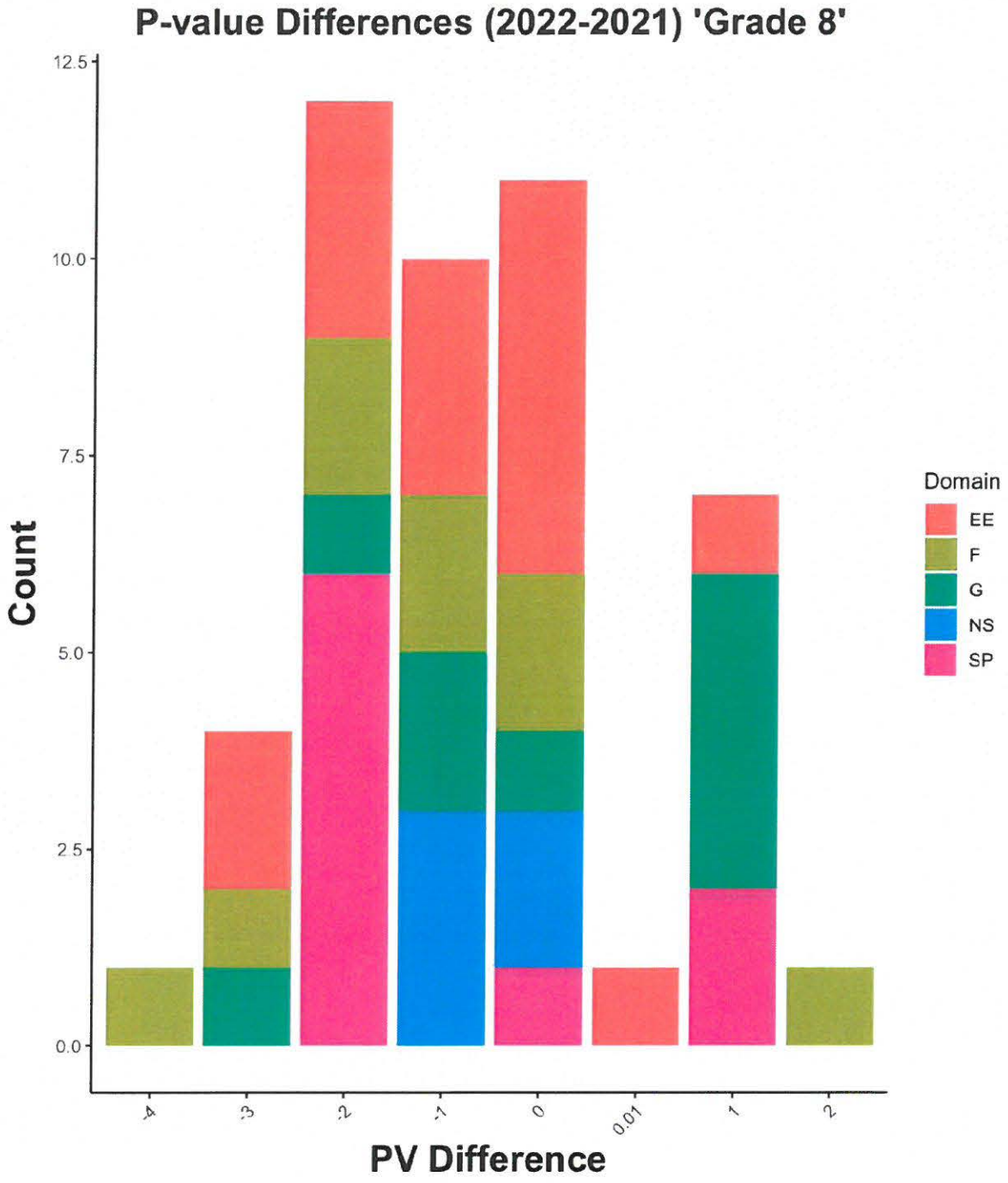

P-value Differences (2022-2021) 'Grade 3'
Count
15
10
5
0
-1
0
0.03
1
2
3
5
6
PV Difference
Domain
G
MD
NBT
NF
OA

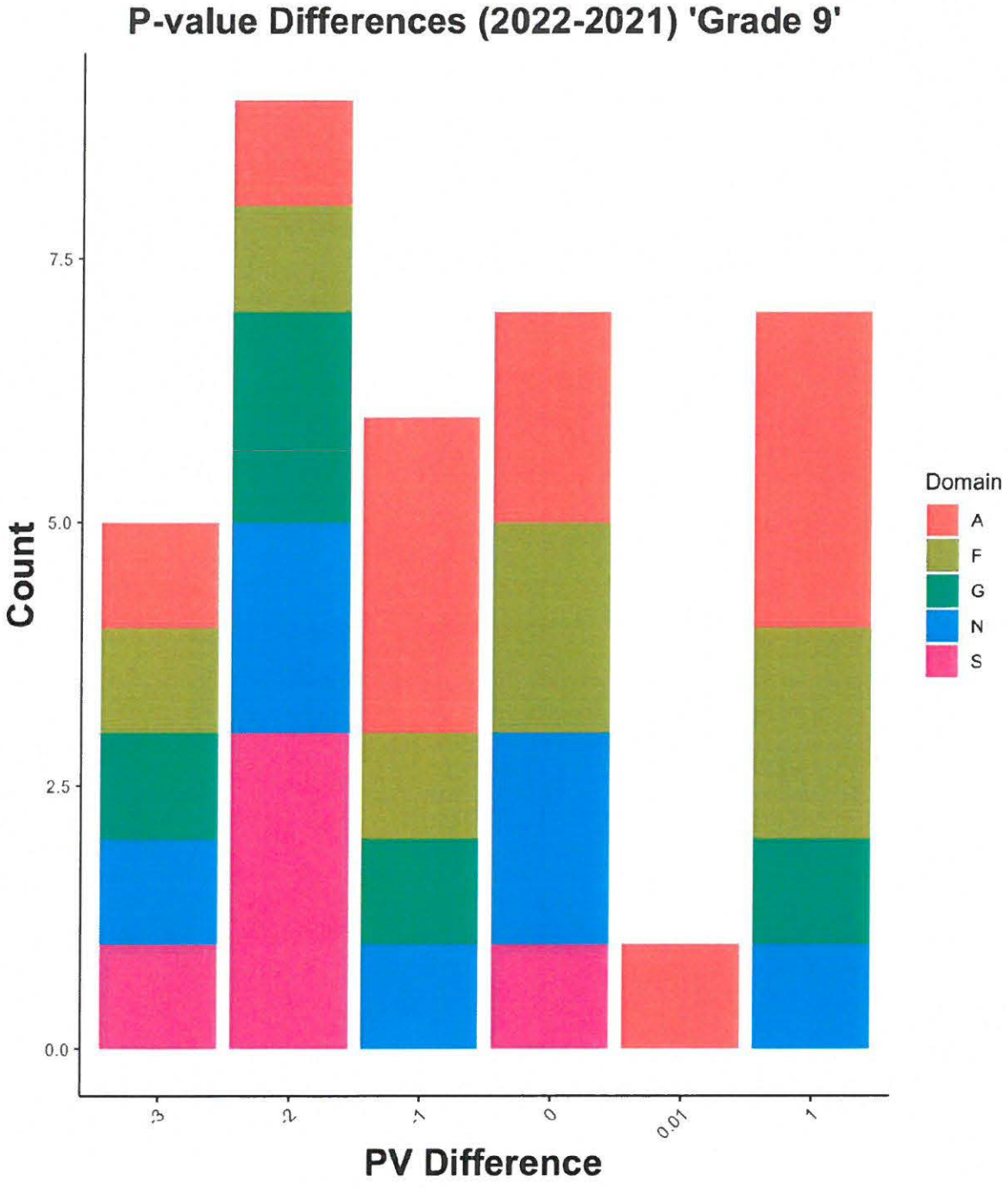

P-value Differences (2022-2021) 'Grade 4'
Count
9
6
3
0
-1
0
0.04
1
2
3
4
PV Difference
Domain
G
MD
NBT
NF
OA

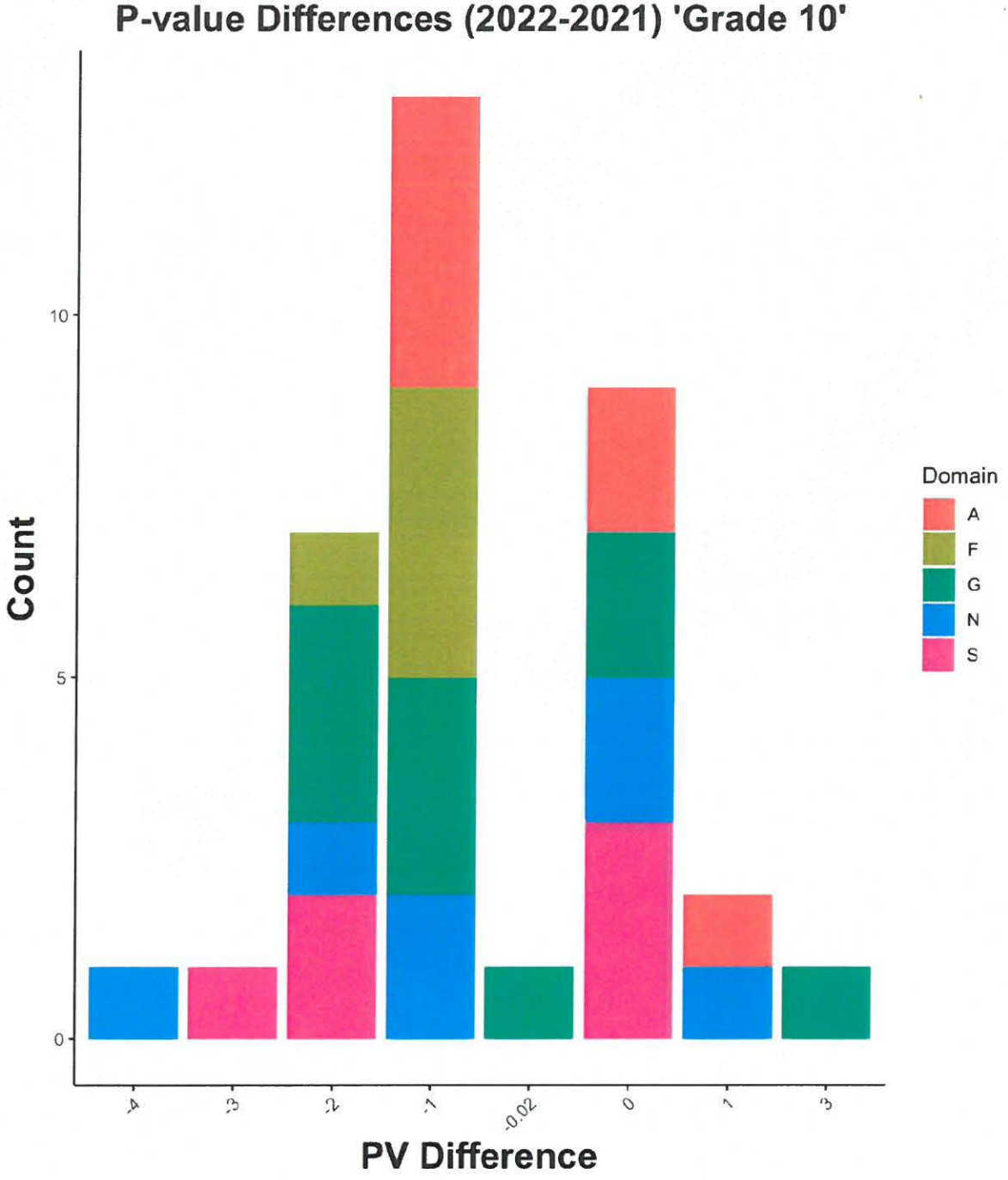

P-value Differences (2022-2021) 'Grade 5'
Count
9
6
3
0
Domain
G
MD
NBT
NF
OA
-3
-2
-1
0
0.03
1
2
3
4
PV Difference

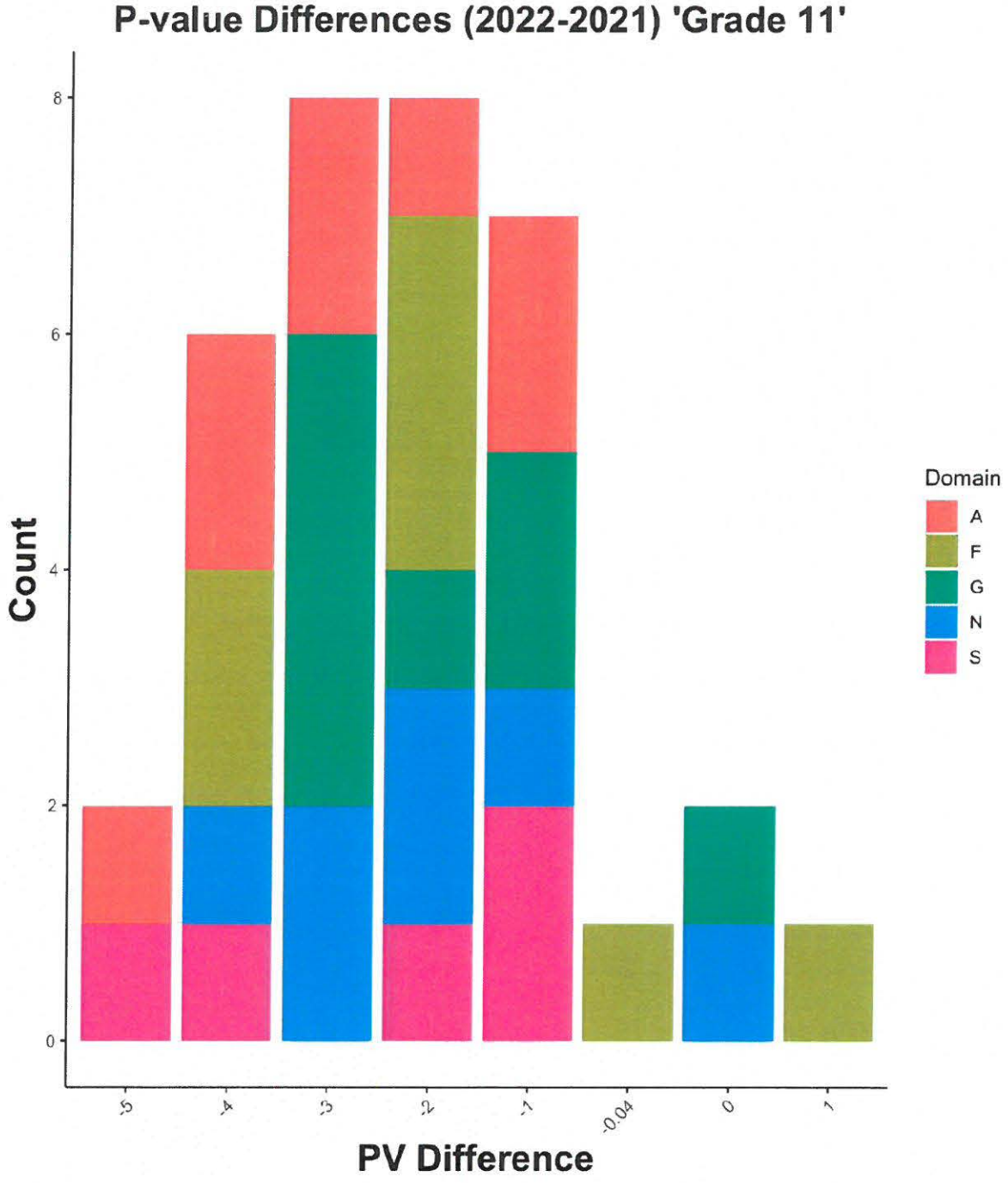

P-value Differences (2022-2021) 'Grade 6'
Count
15
10
5
0
-2
-1
0
1
2
3
4
5
PV Difference
Domain
EE
G
NS
RP
SP


P-value Differences (2022-2021) 'Grade 7'
Count
15
10
5
0
-1
0
1
2
3
4
5
PV Difference
Domain
EE
G
NS
RP
SP


P-value Differences (2022-2021) 'Grade 8'
Count
12.5
10.0
7.5
5.0
2.5
0.0
-.4
-.3
-.2
-.1
0
0.01
.1
.2
PV Difference
Domain
EE
F
G
NS
SP


P-value Differences (2022-2021) 'Grade 9'
Count
7.5
5.0
2.5
0.0
PV Difference
Domain
A
F
G
N
S


P-value Differences (2022-2021) 'Grade 10'
Count
10
5
0
-.4
-.3
-.2
-.1
-0.02
0
.1
.3
PV Difference
Domain
A
F
G
N
S


P-value Differences (2022-2021) 'Grade 11'
Count
8
6
4
2
0
-.5
-.4
-.3
-.2
-.1
-0.04
0
.1
PV Difference
Domain
A
F
G
N
S

Items Representing These Standards Were More Difficult in 2022 than in 2021

Grade	Domain	Standard	Standard Detail
3	MD	3.MD.C.7	Recognize area as additive. Find areas of rectilinear figures by decomposing them into non-overlapping rectangles and adding the areas of the non- overlapping parts, applying this technique to solve real world problems. **(3.MD.C.7)** (DOK 1,2)
3	NF	3.NF.A.1	Understand a fraction $1/b$ as the quantity formed by 1 part when a whole is partitioned into b equal parts; understand a fraction a/b as the quantity formed by a parts of size $1/b$. **(3.NF.A.1)** (DOK 1,2)
3	OA	3.OA.A.2	Interpret whole-number quotients of whole numbers, e.g., interpret 56 ÷ 8 as the number of objects in each share when 56 objects are partitioned equally into 8 shares, or as a number of shares when 56 objects are partitioned into equal shares of 8 objects each. *For example, describe a context in which a number of shares or a number of groups can be expressed as 56 ÷ 8.* **(3.OA.A.2)** (DOK 1,2)
4	MD	4.MD.A.1	Know relative sizes of measurement units within one system of units including km, m, cm; kg, g; lb, oz.; l, ml; hr, min, sec. Within a single system of measurement, express measurements in a larger unit in terms of a smaller unit. Record measurement equivalents in a two-column table. *For example, know that 1 ft is 12 times as long as 1 in. Express the length of a 4 ft snake as 48 in. Generate a conversion table for feet and inches listing the number pairs (1, 12), (2, 24), (3, 36), ...* **(4.MD.A.1)** (DOK 1)
4	NBT	4.NBT.B.6	Find whole-number quotients and remainders with up to four-digit dividends and one- digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models. **(4.NBT.B.6)** (DOK 1,2)
4	NF	4.NF.B.4	Solve word problems involving multiplication of a fraction by a whole number, e.g., by using visual fraction models and equations to represent the problem. *For example, if each person at a party will eat 3/8 of a pound of roast beef, and there will be 5 people at the party, how many pounds of roast beef will be needed? Between what two whole numbers does your answer lie?* **(4.NF.B.4)** (DOK 1,2)
5	G	5.G.A.2	Represent real world and mathematical problems by graphing points in the first quadrant of the coordinate plane, and interpret coordinate values of points in the context of the situation. **(5.G.A.2)** (DOK 1,2)
5	G	5.G.B.3	Understand that attributes belonging to a category of two-dimensional figures also belong to all subcategories of that category. *For example, all rectangles have four right angles and squares are rectangles, so all squares have four right angles.* **(5.G.B.3)** (DOK 1,2)
5	MD	5.MD.A.1	Convert among different-sized standard measurement units within a given measurement system (e.g., convert 5 cm to 0.05 m), and use these conversions in solving multi-step, real world problems. **(5.MD.A.1)** (DOK 1,2)
5	NF	5.NF.A.1	Add and subtract fractions with unlike denominators (including mixed numbers) by replacing given fractions with equivalent fractions in such a way as to produce an equivalent sum or difference of fractions with like denominators. *For example, 2/3 + 5/4 = 8/12 + 15/12 = 23/12. (In general, a/b + c/d = (ad + bc)/bd.)* **(5.NF.A.1)** (DOK 1)
5	NF	5.NF.B.4	Find the area of a rectangle with fractional side lengths by tiling it with unit squares of the appropriate unit fraction side lengths, and show that the area is the same as would be found by multiplying the side lengths. Multiply fractional side lengths to find areas of rectangles, and represent fraction products as rectangular areas. **(5.NF.B.4)** (DOK 1,2)

5	NF	5.NF.B.5	Explaining why multiplying a given number by a fraction greater than 1 results in a product greater than the given number (recognizing multiplication by whole numbers greater than 1 as a familiar case); explaining why multiplying a given number by a fraction less than 1 results in a product smaller than the given number; and relating the principle of fraction equivalence a/b = (n×a)/(n×b) to the effect of multiplying a/b by 1. **(5.NF.B.5)** (DOK 1,2,3)
6	EE	6.EE.B.6	Use variables to represent numbers and write expressions when solving a real-world or mathematical problem; understand that a variable can represent an unknown number, or, depending on the purpose at hand, any number in a specified set. **(6.EE.B.6)** (DOK 1,2)
6	EE	6.EE.B.7	Solve real-world and mathematical problems by writing and solving equations of the form $x + p = q$ and $px = q$ for cases in which p, q and x are all nonnegative rational numbers. **(6.EE.B.7)** (DOK 1,2)
6	EE	6.EE.B.8	Write an inequality of the form $x > c$ or $x < c$ to represent a constraint or condition in a real-world or mathematical problem. Recognize that inequalities of the form $x > c$ or $x < c$ have infinitely many solutions; represent solutions of such inequalities on number line diagrams. **(6.EE.B.8)** (DOK 1,2)
6	G	6.G.A.4	Represent three-dimensional figures using nets made up of rectangles and triangles, and use the nets to find the surface area of these figures. Apply these techniques in the context of solving real-world and mathematical problems. **(6.G.A.4)** (DOK 1,2)
6	NS	6.NS.A.1	Interpret and compute quotients of fractions, and solve word problems involving division of fractions by fractions, e.g., by using visual fraction models and equations to represent the problem. *For example, create a story context for (2/3) ÷ (3/4) and use a visual fraction model to show the quotient; use the relationship between multiplication and division to explain that (2/3) ÷ (3/4) = 8/9 because 3/4 of 8/9 is 2/3. (In general, (a/b) ÷ (c/d) = ad/bc.) How much chocolate will each person get if 3 people share 1/2 lb of chocolate equally? How many 3/4-cup servings are in 2/3 of a cup of yogurt? How wide is a rectangular strip of land with length 3/4 mi and area 1/2 square mi?* **(6.NS.A.1)** (DOK 1,2)
6	NS	6.NS.C.7	Distinguish comparisons of absolute value from statements about order. *For example, recognize that an account balance less than –30 dollars represents a debt greater than 30 dollars.* **(6.NS.C.7)** (DOK 1,2)
6	NS	6.NS.C.8	Solve real-world and mathematical problems by graphing points in all four quadrants of the coordinate plane. Include use of coordinates and absolute value to find distances between points with the same first coordinate or the same second coordinate. **(6.NS.C.8)** (DOK 1,2)
6	RP	6.RP.A.1	Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities. *For example, "The ratio of wings to beaks in the bird house at the zoo was 2:1, because for every 2 wings there was 1 beak." "For every vote candidate A received, candidate C received nearly three votes."* **(6.RP.A.1)** (DOK 1,2)
6	RP	6.RP.A.2	Understand the concept of a unit rate a/b associated with a ratio $a:b$ with $b \neq 0$, and use rate language in the context of a ratio relationship. *For example, "This recipe has a ratio of 3 cups of flour to 4 cups of sugar, so there is 3/4 cup of flour for each cup of sugar." "We paid $75 for 15 hamburgers, which is a rate of $5 per hamburger."*1 **(6.RP.A.2)** (DOK 1,2)
6	RP	6.RP.A.3	Use ratio reasoning to convert measurement units; manipulate and transform units appropriately when multiplying or dividing quantities. **(6.RP.A.3)** (DOK 1,2)
6	SP	6.SP.B.4	Display numerical data in plots on a number line, including dot plots, histograms, and box plots. **(6.SP.B.4)** (DOK 1,2)
6	SP	6.SP.B.5	Relating the choice of measures of center and variability to the shape of the data distribution and the context in which the data were gathered. **(6.SP.B.5)** (DOK 1,2,3)

7	EE	7.EE.B.3	Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies. *For example: If a woman making $25 an hour gets a 10% raise, she will make an additional 1/10 of her salary an hour, or $2.50, for a new salary of $27.50. If you want to place a towel bar 9 3/4 inches long in the center of a door that is 27 1/2 inches wide, you will need to place the bar about 9 inches from each edge; this estimate can be used as a check on the exact computation.* **(7.EE.B.3)** **(DOK 1,2,3)**
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7	G	7.G.A.2	Draw (freehand, with ruler and protractor, and with technology) geometric shapes with given conditions. Focus on constructing triangles from three measures of angles or sides, noticing when the conditions determine a unique triangle, more than one triangle, or no triangle. **(7.G.A.2)** **(DOK 1,2)**
7	G	7.G.B.6	Solve real-world and mathematical problems involving area, volume and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms. **(7.G.B.6)** **(DOK 1,2)**
7	NS	7.NS.A.1	Apply properties of operations as strategies to add and subtract rational numbers. **(7.NS.A.1)** **(DOK 1,2)**
7	NS	7.NS.A.2	Convert a rational number to a decimal using long division; know that the decimal form of a rational number terminates in 0s or eventually repeats. **(7.NS.A.2)** **(DOK 1,2)**
7	NS	7.NS.A.3	Solve real-world and mathematical problems involving the four operations with rational numbers.1 **(7.NS.A.3)** **(DOK 1,2)**
7	RP	7.RP.A.1	Compute unit rates associated with ratios of fractions, including ratios of lengths, areas and other quantities measured in like or different units. *For example, if a person walks 1/2 mile in each 1/4 hour, compute the unit rate as the complex fraction 1/2/1/4 miles per hour, equivalently 2 miles per hour.* **(7.RP.A.1)** **(DOK 1,2)**
7	RP	7.RP.A.2	Explain what a point (x, y) on the graph of a proportional relationship means in terms of the situation, with special attention to the points (0, 0) and (1, r) where r is the unit rate. **(7.RP.A.2)** **(DOK 1,2)**
7	RP	7.RP.A.2	Explain what a point (x, y) on the graph of a proportional relationship means in terms of the situation, with special attention to the points (0, 0) and (1, r) where r is the unit rate. **(7.RP.A.2)** **(DOK 1,2)**
7	SP	7.SP.A.1	Understand that statistics can be used to gain information about a population by examining a sample of the population; generalizations about a population from a sample are valid only if the sample is representative of that population. Understand that random sampling tends to produce representative samples and support valid inferences. **(7.SP.A.1)** **(DOK 2)**
7	SP	7.SP.B.4	Use measures of center and measures of variability for numerical data from random samples to draw informal comparative inferences about two populations. *For example, decide whether the words in a chapter of a seventh-grade science book are generally longer than the words in a chapter of a fourth-grade science book.* **(7.SP.B.4)** **(DOK 2,3)**

7	SP	7.SP.C.6	Approximate the probability of a chance event by collecting data on the chance process that produces it and observing its long-run relative frequency, and predict the approximate relative frequency given the probability. *For example, when rolling a number cube 600 times, predict that a 3 or 6 would be rolled roughly 200 times, but probably not exactly 200 times.* **(7.SP.C.6)** **(DOK 2,3)**
8	EE	8.EE.A.1	Know and apply the properties of integer exponents to generate equivalent numerical expressions. *For example, 32 × 3–5 = 3–3 = 1/33 = 1/27* . **(8.EE.A.1)** **(DOK 1)**
8	EE	8.EE.A.2	Use square root and cube root symbols to represent solutions to equations of the form x2 = p and x3 = p, where p is a positive rational number. Evaluate square roots of small perfect squares and cube roots of small perfect cubes. Know that √2 is irrational. **(8.EE.A.2)** **(DOK 1)**
8	EE	8.EE.A.3	Use numbers expressed in the form of a single digit times an integer power of 10 to estimate very large or very small quantities, and to express how many times as much one is than the other. *For example, estimate the population of the United States as 3 × 108 and the population of the world as 7 × 109, and determine that the world population is more than 20 times larger.* **(8.EE.A.3)** **(DOK 1,2)**
8	EE	8.EE.A.4	Perform operations with numbers expressed in scientific notation, including problems where both decimal and scientific notation are used. Use scientific notation and choose units of appropriate size for measurements of very large or very small quantities (e.g., use millimeters per year for seafloor spreading). Interpret scientific notation that has been generated by technology. **(8.EE.A.4)** **(DOK 1,2)**
8	EE	8.EE.B.5	Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways. *For example, compare a distance-time graph to a distance-time equation to determine which of two moving objects has greater speed.* **(8.EE.B.5)** **(DOK 1,2,3)**
8	EE	8.EE.B.6	Use similar triangles to explain why the slope m is the same between any two distinct points on a non-vertical line in the coordinate plane; derive the equation $y = mx$ for a line through the origin and the equation $y = mx + b$ for a line intercepting the vertical axis at b . **(8.EE.B.6)** **(DOK 1,2,3)**
8	EE	8.EE.C.7	Solve linear equations with rational number coefficients, including equations whose solutions require expanding expressions using the distributive property and collecting like terms. **(8.EE.C.7)** **(DOK 1,2)**
8	EE	8.EE.C.7	Solve linear equations with rational number coefficients, including equations whose solutions require expanding expressions using the distributive property and collecting like terms. **(8.EE.C.7)** **(DOK 1,2)**
8	EE	8.EE.C.8	Solve real-world and mathematical problems leading to two linear equations in two variables. *For example, given coordinates for two pairs of points, determine whether the line through the first pair of points intersects the line through the second pair.* **(8.EE.C.8)** **(DOK 1,2,3)**
8	F	8.F.A.1	Understand that a function is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting of an input and the corresponding output.1 **(8.F.A.1)** **(DOK 1,2)**
8	F	8.F.A.2	Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a linear function represented by a table of values and a linear function represented by an algebraic expression, determine which function has the greater rate of change.* **(8.F.A.2)** **(DOK 1,2)**
8	F	8.F.A.3	Interpret the equation $y = mx + b$ as defining a linear function, whose graph is a straight line; give examples of functions that are not linear. *For example, the function A = s2 giving the area of a square as a function of its side length is not linear because its graph contains the points (1,1), (2,4) and (3,9), which are not on a straight line.* **(8.F.A.3)** **(DOK 1,2)**
8	F	8.F.B.4	Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (x, y) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values. **(8.F.B.4)** **(DOK 1,2,3)**

8	G	8.G.A.1	"Verify experimentally the properties of rotations, reflections, and translations: a. Lines are taken to lines, and line segments to line segments of the same length. b. Angles are taken to angles of the same measure. c. Parallel lines are taken to parallel lines. (8.G.A.1) (DOK 2)"
8	G	8.G.A.3	Describe the effect of dilations, translations, rotations, and reflections on two- dimensional figures using coordinates. **(8.G.A.3)** (DOK 1,2)
8	G	8.G.A.5	Use informal arguments to establish facts about the angle sum and exterior angle of triangles, about the angles created when parallel lines are cut by a transversal, and the angle-angle criterion for similarity of triangles. *For example, arrange three copies of the same triangle so that the sum of the three angles appears to form a line, and give an argument in terms of transversals why this is so.* **(8.G.A.5)** (DOK 1,2,3)
8	G	8.G.C.9	Know the formulas for the volumes of cones, cylinders, and spheres and use them to solve real-world and mathematical problems. **(8.G.C.9)** (DOK 1,2)
8	NS	8.NS.A.1	Know that numbers that are not rational are called irrational. Understand informally that every number has a decimal expansion; for rational numbers show that the decimal expansion repeats eventually, and convert a decimal expansion which repeats eventually into a rational number. **(8.NS.A.1)** (DOK 1)
8	NS	8.NS.A.2	Use rational approximations of irrational numbers to compare the size of irrational numbers, locate them approximately on a number line diagram, and estimate the value of expressions (e.g., π2). *For example, by truncating the decimal expansion of √2, show that √2 is between 1 and 2, then between 1.4 and 1.5, and explain how to continue on to get better approximations.* **(8.NS.A.2)** (DOK 1,2)
8	SP	8.SP.A.1	Construct and interpret scatter plots for bivariate measurement data to investigate patterns of association between two quantities. Describe patterns such as clustering, outliers, positive or negative association, linear association, and nonlinear association. **(8.SP.A.1)** (DOK 1,2,3)
8	SP	8.SP.A.2	Know that straight lines are widely used to model relationships between two quantitative variables. For scatter plots that suggest a linear association, informally fit a straight line, and informally assess the model fit by judging the closeness of the data points to the line. **(8.SP.A.2)** (DOK 1,2)
8	SP	8.SP.A.3	Use the equation of a linear model to solve problems in the context of bivariate measurement data, interpreting the slope and intercept. *For example, in a linear model for a biology experiment, interpret a slope of 1.5 cm/hr as meaning that an additional hour of sunlight each day is associated with an additional 1.5 cm in mature plant height.* **(8.SP.A.3)** (DOK 1,2)
8	SP	8.SP.A.4	Understand that patterns of association can also be seen in bivariate categorical data by displaying frequencies and relative frequencies in a two-way table. Construct and interpret a two-way table summarizing data on two categorical variables collected from the same subjects. Use relative frequencies calculated for rows or columns to describe possible association between the two variables. *For example, collect data from students in your class on whether or not they have a curfew on school nights and whether or not they have assigned chores at home. Is there evidence that those who have a curfew also tend to have chores?* **(8.SP.A.4)** (DOK 1,2,3)
9	A	A-APR.A.1	Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials. **(A-APR.A.1)** (DOK 1)
9	A	A-CED.A.1	Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions. **(A-CED.A.1)** (DOK 1,2)
9	A	A-CED.A.2	Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. **(A-CED.A.2)** (DOK 1,2)
9	A	A-CED.A.4	Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. For example, rearrange Ohm's law V = IR to highlight resistance R. **(A-CED.A.4)** (DOK 1)

9	A	A-REI.B.4	Solve quadratic equations by inspection (e.g., for x2 = 49), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as a ± bi for real numbers a and b. **(A-REI.B.4)** (DOK 1,2,3)
9	A	A-SSE.A.1	Interpret complicated expressions by viewing one or more of their parts as a single entity. For example, interpret P(1+r)n as the product of P and a factor not depending on P. **(A-SSE.A.1)** (DOK 1,2)
9	F	F-BF.A.1	Write a function that describes a relationship between two quantities.* a.Determine an explicit expression, a recursive process, or steps for calculation from a context. b. ACombine standard function types using arithmetic operations. For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model. c. (+) Compose functions. For example, if T(y) is the temperature in the atmosphere as a function of height, and h(t) is the height of a weather balloon as a function of time, then T(h(t)) is the temperature at the location of the weather balloon as a function of time.
9	F	F-BF.B.3	Identify the effect on the graph of replacing f(x) by f(x) + k, k f(x), f(kx), and f(x + k) for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.
9	F	F-IF.B.5	Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function h(n) gives the number of person- hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.* ★ **(F-IF.B.5)** (DOK 1,2) (DOK 1,2)
9	F	F-LE.A.2	Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table). **(F-LE.A.2)** (DOK 1,2)
9	G	G-GMD.B.4	4. Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects. **(G-GMD.B.4)** (DOK 1,2)
9	G	G-SRT.A.2	Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides. (G-SRT.A.2) (DOK 1,2)
9	N	N-Q.A.1	Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays. **(N-Q.A.1)** (DOK 1,2)
9	N	N-Q.A.2	Define appropriate quantities for the purpose of descriptive modeling. **(N-Q.A.2)** (DOK 1,2)
9	N	N-RN.A.1	Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents. *For example, we define 51/3 to* be the cube root of 5 because we want (51/3)3 = 5(1/3)3 to hold, so (51/3)3 must equal 5. **(N-RN.A.1)** (DOK 1,2)
9	S	S-CP.A.1	Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events ("or," "and," "not") **(S-CP.A.1)** (DOK 1,2)
9	S	S-CP.B.7	Apply the Addition Rule, $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$, and interpret the answer in terms of the model. **(S-CP.B.7)** (DOK 1,2)
9	S	S-ID.A.2	Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets. **(S-ID.A.2)** (DOK 1,2)

9	S	S-ID.B.5	Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies). Recognize possible associations and trends in the data. **(S-ID.B.5)** (DOK 1,2)
10	A	A-APR.A.1	Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials. **(A-APR.A.1)** (DOK 1)
10	A	A-APR.D.6	Rewrite simple rational expressions in different forms; write $a(x)/b(x)$ in the form $q(x) + r(x)/b(x)$, where $a(x)$, $b(x)$, $q(x)$, and $r(x)$ are polynomials with the degree of $r(x)$ less than the degree of $b(x)$, using inspection, long division, or, for the more complicated examples, a computer algebra system. **(A-APR.D.6)** (DOK 1,2)
10	A	A-CED.A.1	Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions. **(A-CED.A.1)** (DOK 1,2)
10	A	A-REI.A.1	Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method. **(A-REI.A.1)** (DOK 1,2,3)
10	A	A-REI.B.3	Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters. **(A-REI.B.3)** (DOK 1)
10	F	F-BF.A.1	"Write a function that describes a relationship between two quantities.* a.Determine an explicit expression, a recursive process, or steps for calculation from a context. b. ACombine standard function types using arithmetic operations. For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model. c. (+) Compose functions. For example, if T(y) is the temperature in the atmosphere as a function of height, and h(t) is the height of a weather balloon as a function of time, then T(h(t)) is the temperature at the location of the weather balloon as a function of time."
10	F	F-IF.A.2	Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context. **(F-IF.A.2)** (DOK 1,2)
10	F	F-IF.B.5	Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function h(n) gives the number of person- hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.* ★ **(F-IF.B.5)** (DOK 1,2) (DOK 1,2)
10	F	F-IF.B.6	Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.★ **(F-IF.B.6)**
10	F	F-LE.A.2	Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table). **(F-LE.A.2)** (DOK 1,2)
10	G	G-C.A.3	Construct the inscribed and circumscribed circles of a triangle, and prove properties of angles for a quadrilateral inscribed in a circle. **(G-C.A.3)** (DOK 2,3)
10	G	G-CO.A.1	Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc. **(G-CO.A.1)** (DOK 1)
10	G	G-CO.A.3	Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and reflections that carry it onto itself. **(G-CO.A.3)** (DOK 1,2)
10	G	G-CO.B.8	Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence in terms of rigid motions. **(G-CO.B.8)** (DOK 2,3)
10	G	G-GPE.B.5	Prove the slope criteria for parallel and perpendicular lines and use them to solve geometric problems (e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point). **(G-GPE.B.5)** (DOK 1,2)

10	G	G-SRT.A.2	Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides. **(G-SRT.A.2)** (DOK 1,2)
10	G	G-SRT.C.8	Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.★ **(G-SRT.C.8)** (DOK 1,2)
10	N	N-Q.A.2	Define appropriate quantities for the purpose of descriptive modeling. **(N-Q.A.2)** (DOK 1,2)
10	N	N-Q.A.3	Choose a level of accuracy appropriate to limitations on measurement when reporting quantities. **(N-Q.A.3)** (DOK 1,2)
10	N	N-RN.A.2	Rewrite expressions involving radicals and rational exponents using the properties of exponents. **(N-RN.A.2)** (DOK 1)
10	S	S-CP.A.1	Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events ("or," "and," "not") **(S-CP.A.1)** (DOK 1,2)
10	S	S-ID.A.1	Represent data with plots on the real number line (dot plots, histograms, and box plots).**(S-ID.A.1)** (DOK 1,2)
10	S	S-ID.A.2	Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets. **(S-ID.A.2)** (DOK 1,2)
10	S	S-ID.B.5	Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies). Recognize possible associations and trends in the data. **(S-ID.B.5)** (DOK 1,2)
11	A	A-APR.A.1	1. Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials. **(A-APR.A.1)** (DOK 1)
11	A	A-CED.A.1	Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions. **(A-CED.A.1)** (DOK 1,2)
11	A	A-REI.C.6	Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables. **(A-REI.C.6)** (DOK 1,2)
11	A	A-SSE.A.2	**Use the structure of an expression to identify ways to rewrite it. For example, see x4 – y4 as (x2)2 – (y2)2, thus recognizing it as a difference of squares that can be factored as (x2 – y2)(x2** + y2). **(A-SSE.A.2)** (DOK 1,2)
11	F	F-IF.A.2	Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context. **(F-IF.A.2)** (DOK 1,2)
11	F	F-IF.B.4	For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.* ★ **(F-IF.B.4)** (DOK 1,2)
11	F	F-IF.C.7	Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude. **(F-IF.C.7)** (DOK 1,2)
11	F	F-LE.A.2	Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or tw
11	G	G-C.A.2	Identify and describe relationships among inscribed angles, radii, and chords. Include the relationship between central, inscribed, and circumscribed angles; inscribed angles on a diameter are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects the circle. **(G-C.A.2)** (DOK 1,2)

11	G	G-CO.C.10	Prove theorems about triangles. *Theorems include: measures of interior angles of a triangle sum to 180°; base angles of isosceles triangles are congruent; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point.* **(G-CO.C.10)** (DOK 3)
11	G	G-GPE.B.6	Find the point on a directed line segment between two given points that partitions the segment in a given ratio. **(G-GPE.B.6)** (DOK 1,2)
11	G	G-GPE.B.7	Use coordinates to compute perimeters of polygons and areas of triangles and rectangles, e.g., using the distance formula.★ **(G-GPE.B.7)** (DOK 1,2)
11	G	G-SRT.C.7	Explain and use the relationship between the sine and cosine of complementary angles. **(G-SRT.C.7)** (DOK 1,2)
11	N	N-CN.C.7	Solve quadratic equations with real coefficients that have complex solutions. **(N-CN.C.7)** (DOK 1)
11	N	N-Q.A.1	Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays. **(N-Q.A.1)** (DOK 1,2)
11	N	N-Q.A.2	Define appropriate quantities for the purpose of descriptive modeling. **(N-Q.A.2)** (DOK 1,2)
11	N	N-Q.A.3	Choose a level of accuracy appropriate to limitations on measurement when reporting quantities. **(N-Q.A.3)** (DOK 1,2)
11	N	N-RN.A.2	Rewrite expressions involving radicals and rational exponents using the properties of exponents. **(N-RN.A.2)** (DOK 1)
11	S	S-CP.A.1	Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events ("or," "and," "not") **(S-CP.A.1)** (DOK 1,2)
11	S	S-CP.B.9	(+) Use permutations and combinations to compute probabilities of compound events and solve problems. **(S-CP.B.9)** (DOK 1,2)
11	S	S-ID.A.2	Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets. **(S-ID.A.2)** (DOK 1,2)
11	S	S-ID.A.4	Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets, and tables to estimate areas under the normal curve. **(S-ID.A.4)** (DOK 1,2)
11	S	S-ID.B.6	Represent data on two quantitative variables on a scatter plot, and describe how the variables are related. a.Fit a function to the data; use functions fitted to data to solve problems in the context of the data. Use given functions or choose a function suggested by the context. Emphasize linear, quadratic, and exponential models. b.Informally assess the fit of a function by plotting and analyzing residuals. c. Fit a linear function for a scatter plot that suggests a linear association.